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Random Differential Equations Associated with Accretive Operators

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INTRODUCTION

Extensive studies on differential equations with (nonlinear) accretive operators in Banach or Hilbert spaces have been obtained by many authors (cf. Brézis [5], Barbu [1], Crandall [8] and their references). On the other hand, the theory of random equations have attracted much attention (cf. Bharucha-Reid [2, 3, 4], Hanš [12, 13, 14], Kampé de Fériet [20], Nashed and Salehi [25], Kannan and Salehi [21], Itoh [17, 18, 19] and their references).

In this paper we deal with random differential equations associated with continuous nonlinear accretive operators in Banach or Hilbert spaces. In section 2 we show the existence of solutions of random differential equations with time independent accretive operators in Banach spaces, while in section 3 with time dependent accretive operators in Hilbert spaces. Then in section 4, using the above results we prove the existence of periodic solutions for some types of random differential equations considered in sections 3 and 4.

1. PRELIMINARIES

Let X be a Banach space and X^* be its dual space with $\langle\cdot,\cdot\rangle$ the duality pairing between X and X^*. We use the convention that $(\cdot,\cdot) = \mathrm{Re}\langle\cdot,\cdot\rangle$ if X is complex or $(\cdot,\cdot) = \langle\cdot,\cdot\rangle$ if X is real, where $\mathrm{Re}\, z$ is the real part of a complex number z. Let D be a subset of X. A mapping $S: D \to X$ is said to be *k-Lipschitz* $(k \geqslant 0)$ if $\| Sx - Sy \| \leqslant k \| x - y \|$ for for every $x, y \in D$. A k-Lipschitz mapping S is called (k-) *contraction* or *nonexpansive* if $k < 1$ or $k = 1$ respectively. A mapping $S: [0, \infty) \times D \to D$ with the following properties is said to be a *semigroup of type k* on D:

(i) $S(t)\, S(s)x = S(t+s)x$ for $t, s \geqslant 0$ and $x \in D$;
(ii) $\| S(t)x - S(t)y \| \leqslant e^{kt} \| x - y \|$ for $t \geqslant 0$ and $x, y \in D$;
(iii) $\lim_{t \downarrow 0} S(t)x = S(0)x = x$ for $x \in D$.

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For each $x \in X$, denote $F(x) = \{x^* \in X^*: \langle x, x^* \rangle = \| x \|^2 = \| x^* \|^2\}$, then $F(x)$ is nonempty and weakly* compact. For any $x, y \in X$, define $\langle x, y \rangle_s = \sup\{(x, y^*): y^* \in F(y)\}$, $\langle x, y \rangle_i = \inf\{(x, y^*): y^* \in F(y)\}$,

$$[x, y]_+ = \lim_{\lambda \downarrow 0}\{(\| x + \lambda y \| - \| x \|)/\lambda\},$$

and

$$[x, y]_- = \lim_{\lambda \uparrow 0}\{(\| x + \lambda y \| - \| x \|)/\lambda\},$$

respectively. A mapping $A: D \to X$ is said to be *accretive* if A satisfies one (all) of the following equivalent conditions (cf. Crandall [8] and Crandall and Liggett [9]):

(i) $[x - y, Ax - Ay]_+ \geqslant 0$ for $x, y \in D$;

(ii) $\langle Ax - Ay, x - y \rangle_s \geqslant 0$ for $x, y \in D$;

(iii) For any $\lambda > 0$, $(I + \lambda A)^{-1}$ is a (singlevalued) nonexpansive mapping from $R(I + \lambda A)$ (the range of $I + \lambda A$) into D, where I is the identity mapping of X.

If X is a Hilbert space, then an accretive operator A is also called *monotone*, and A is monotone if and only if $(Ax - Ay, x - y) \geqslant 0$ for $x, y \in D$, where $(\cdot, \cdot)$ is induced by the inner product $\langle \cdot, \cdot \rangle$ on X by the same way as above.

We always denote by $(\Omega, \mathscr{A})$ a measurable space. A mapping $S: \Omega \to X$ is said to be *measurable* if for any closed subset C of X, $S^{-1}(C) = \{\omega \in \Omega: S(\omega) \in C\} \in \mathscr{A}$. If X is separable, other definitions of measurability of mappings $S: \Omega \to X$ are equivalent to the above (cf. Bharucha-Reid [3, pp. 14–16] and Hille and Phillips [15, pp. 72–73]). A mapping $S: \Omega \times D \to X$ is said to be a *random operator* if for each $x \in D$, $S(\cdot)x: \Omega \to X$ is measurable. A random operator S is said to be continuous (accretive, etc.) if for any $\omega \in \Omega$, $S(\omega): D \to X$ is continuous (accretive, etc.).

In this paper derivatives are strong derivatives and integrals are Bochner integrals (cf. Hille and Phillips [15, Chapter III]). Let $[0, T]$ be a finite closed interval of the real line R. Denote by $C([0, T], X)$ the Banach space of all continuous mappings $v: [0, T] \to X$ with the norm $\| v \|_\infty = \max\{\| v(t) \|: 0 \leqslant t \leqslant T\}$. Also denote by $C_1([0, T], X)$ the set of all continuously differentiable mappings v: $[0, T] \to X$. A mapping $u: [0, T] \times \Omega \to X$ is said to satisfy *condition* (C, Ω) (or (C_1, Ω)) if for each $t \in [0, T]$, $u(t, \cdot)$ is measurable and for each $\omega \in \Omega$, $u(\cdot, \omega)$ is continuous (or continuously differentiable). For the interval $[0, \infty)$ or other interval of R, we also adopt similar definitions.

2. Random Differential Equations in Banach Spaces

First we give a fundamental result related to inverses of random operators which was essentially obtained by Hanš [13] (cf. Nashed and Salehi [25]). We include the proof for completeness.

LEMMA 2.1. *Let X be a separable Banach space and Y be a Banach space. Let T: $\Omega \times X \to Y$ be a continuous random operator such that for any $\omega \in \Omega$, $T(\omega)$ has the continuous inverse $T(\omega)^{-1}$. Then the mapping S: $\Omega \times Y \to X$ defined by $S(\omega)y = T(\omega)^{-1}(y)(\omega \in \Omega, y \in Y)$ is a continuous random operator.*

Proof. Fix an element y of Y. For each closed subset C of X, there exists a countable dense subset $\{x_i\}$ of C. Then it follows that

$$\{\omega \in \Omega : S(\omega)y \in C\} = \bigcup_{x \in C} \{\omega \in \Omega : S(\omega)y = x\}$$

$$= \bigcap_{n=1}^{\infty} \bigcup_{i=1}^{\infty} \{\omega \in \Omega : \| y - T(\omega)x_i \| < 1/n\} \in \mathscr{A}.$$

Hence S is a random operator. Q.E.D.

Now we present a theorem which is concerned with semigroups and solutions of random differential equations with time independent accretive operators. In the deterministic case Martin [23] and Webb [27] treated similar differential equations (cf. Barbu [1] and Brézis [5]).

THEOREM 2.2. *Let X be a separable Banach space and A: $\Omega \times X \to X$ be a continuous random operator. Suppose there exists a function k: $\Omega \to R$ such that $\sup\{k(\omega): \omega \in \Omega\} = M < \infty$ and for each $\omega \in \Omega$, $A(\omega) + k(\omega)I$ is accretive. Then the followings hold*:

(i) *There exists a mapping S: $[0, \infty) \times \Omega \times X \to X$ such that for each $\omega \in \Omega$, $S(\cdot, \omega)$ is a semigroup of type $k(\omega)$ on X and for each $t \geqslant 0$ and $x \in X$, $S(t, \cdot)x$ is measurable*;

(ii) *If v: $\Omega \to X$ is measurable, then the mapping u: $[0, \infty) \times \Omega \to X$ defined by $u(t, \omega) = S(t, \omega)\, v(\omega)$ $(t \geqslant 0, \omega \in \Omega)$ satisfies condition (C_1, Ω) and this u is the unique mapping such that for any $\omega \in \Omega$, $u(0, \omega) = v(\omega)$ and*

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) = 0 \quad (t \geqslant 0).$$

Proof. By Martin [24, Theorem 6] and the hypotheses, for sufficiently small $\lambda > 0$, the range of $I + \lambda A(\omega)$ is equal to X for all $\omega \in \Omega$. Hence the mapping J: $\Omega \times X \to X$ by $J_\lambda(\omega)x = (I + \lambda A(\omega))^{-1}x$ $(\omega \in \Omega,\ x \in X)$ is a continuous (Lipschitz) random operator by Crandall and Liggett [9, Lemma 1.2] and Lemma 1.1. Define S: $[0, \infty) \times \Omega \times X \to X$ by

$$S(t, \omega)x = \lim_{n \to \infty} J_{t/n}(\omega)^n x \qquad \text{for} \quad t \geqslant 0, \omega \in \Omega \text{ and } x \in X,$$

then for each $\omega \in \Omega$, $S(\cdot, \omega)$ is a semigroup of type $k(\omega)$ on X by Crandall and Liggett [9, Theorem I] and for each $t \geqslant 0$ and $x \in X$, $S(t, \cdot)x$ is measurable.

Thus (i) holds. For $\omega \in \Omega$ and $x \in X$, $S(\cdot, \omega)x$ is continuously differentiable and

$$dS(t, \omega)x/dt + A(\omega) S(t, \omega)x = 0 \qquad (t \geqslant 0)$$

(cf. Crandall and Liggett [9, pp. 282–283]). Moreover, if $v: \Omega \to X$ is measurable, then for each $t \geqslant 0$, $S(t, \cdot)\, v(\cdot)$ is measurable (cf. Himmelberg [16, Theorem 6.5]), which and [9, Theorem II] together yield (ii). Q.E.D.

Using the above results, we extend (ii) of Theorem 1.2 in the following.

THEOREM 2.3. *Let X be a separable Banach space, $A: \Omega \times X \to X$ be a continuous random operator and $f: [0, T] \times \Omega \to X$ be a mapping satisfying condition (C, Ω). Suppose there exists a function $k: \Omega \to R$ such that* $\sup\{k(\omega): \omega \in \Omega\} = M < \infty$ *and for any $\omega \in \Omega$, $A(\omega) + k(\omega)I$ is accretive. Then for any measurable mapping $v: \Omega \to X$, there exists a unique mapping $u: [0, T] \times \Omega \to X$ satisfying condition (C_1, Ω) such that for any $\omega \in \Omega$, $u(0, \omega) = v(\omega)$ and*

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

Proof. For any positive integer n, consider a partition $\{t_i^n\}_{i=0}^{2^n}$ of $[0, T]$ defined by

$$0 = t_0^n < t_1^n < \cdots < t_{2^n}^n = T, \qquad t_i^n - t_{i-1}^n = T/2^n$$

$(i = 1, 2,..., 2^n)$. Let $f_n : [0, T] \times \Omega \to X$ be a mapping defined by $f_n(t, \omega) = f(t_i^n, \omega)$ $(\omega \in \Omega,\ t_{i-1}^n \leqslant t \leqslant t_i^n,\ i = 1, 2,..., 2^n)$. Define $A_{ni} : \Omega \times X \to X$ by $A_{ni}(\omega)x = A(\omega)x - f(t_i^n, \omega)$ $(\omega \in \Omega,\ x \in X)$, then A_{ni} is a continuous random operator such that for each $\omega \in \Omega$, $A_{ni}(\omega) + k(\omega)I$ is accretive. Each A_{ni} has a mapping $S_{ni} : [0, \infty) \times \Omega \times X \to X$ as in Theorem 1.2. Let $u_n : [0, T] \times \Omega \to X$ be a mapping inductively defined by

$$u_n(0, \omega) = v(\omega)$$

and

$$u_n(t, \omega) = S_{ni}(t - t_{i-1}^n, \omega)\, u_n(t_{i-1}^n, \omega)$$

$(\omega \in \Omega,\ t_{i-1}^n \leqslant t \leqslant t_i^n)$, then u_n satisfies condition (C, Ω) and for any $\omega \in \Omega$, $u_n(\cdot, \omega)$ is differentiable at $t \neq t_i^n$ $(i = 1, 2,..., 2^n)$. Let $m > n$, then for any fixed $\omega \in \Omega$, $\| u_m(\cdot, \omega) - u_n(\cdot, \omega)\|$ is absolutely continuous, hence differentiable a.e. on $[0, T]$. For a.e. $t \in [0, T]$, choose i, j such that $t_{i-1}^n \leqslant t_{j-1}^m \leqslant t \leqslant t_j^m \leqslant t_i^n$, then we have (cf. Crandall [8, p. 138]).

$$\begin{aligned}
&d\| u_m(t, \omega) - u_n(t, \omega)\|/dt \\
&\quad = [u_m(t, \omega) - u_n(t, \omega), du_m(t, \omega)/dt - du_n(t, \omega)/dt]_- \\
&\quad = [u_m(t, \omega) - u_n(t, \omega), -A(\omega)\, u_m(t, \omega) + f(t_j^m, \omega) + A(\omega)\, u_n(t, \omega) - f(t_i^n, \omega)]_- \\
&\quad \leqslant -[u_m(t, \omega) - u_n(t, \omega), (A(\omega) + k(\omega)I)\, u_m(t, \omega) - (A(\omega) + k(\omega)I)\, u_n(t, \omega)]_+ \\
&\qquad + [u_m(t, \omega) - u_n(t, \omega), k(\omega)(u_m(t, \omega) - u_n(t, \omega)]_- \\
&\qquad + [u_m(t, \omega) - u_n(t, \omega), f_m(t, \omega) - f_n(t, \omega)]_+ \\
&\quad \leqslant k(\omega)\, \| u_m(t, \omega) - u_n(t, \omega)\| + \| f_m(t, \omega) - f_n(t, \omega)\|.
\end{aligned}$$

Denote $L(\omega) = \min\{0, k(\omega)\}$, then the above inequality shows that for any $t \in [0, T]$,

$$\begin{aligned}
&\| u_m(t, \omega) - u_n(t, \omega)\| \\
&\quad \leqslant e^{(k(\omega)-L(\omega))T} \left\{\int_0^T \|f_m(t, \omega) - f_n(t, \omega)\| \, dt\right\} \\
&\quad \leqslant e^{(k(\omega)-L(\omega))T} \\
&\qquad \times \left\{\int_0^T \|f_m(t, \omega) - f(t, \omega)\| \, dt + \int_0^T \|f_n(t, \omega) - f(t, \omega)\| \, dt\right\}.
\end{aligned}$$

This implies that for each $\omega \in \Omega$, $\{u_n(t, \omega)\}$ converges to some $u(t, \omega)$ uniformly in t on $[0, T]$. The mapping u: $[0, T] \times \Omega \to X$ satisfies condition (C, Ω). For any $\omega \in \Omega$, $t \in [0, T]$ and n, take i so that $t_{i-1}^n \leqslant t \leqslant t_i^n$, then it follows that

$$\begin{aligned}
u_n(t, \omega) &= v(\omega) + u_n(t, \omega) - u_n(t_{i-1}^n, \omega) \\
&\quad + \sum_{j=1}^{i-1} \{u_n(t_j^n, \omega) - u_n(t_{j-1}^n, \omega)\} \\
&= v(\omega) - \int_{t_{i-1}^n}^t A_{ni}(\omega) \, u_n(s, \omega) \, ds - \sum_{j=1}^{i-1} \int_{t_{j-1}^n}^{t_j^n} A_{nj}(\omega) \, u_n(s, \omega) \, ds \\
&= v(\omega) - \int_{t_{i-1}^n}^t \{A(\omega) \, u_n(s, \omega) - f_n(s, \omega)\} \, ds \\
&\quad - \sum_{j=1}^{i-1} \int_{t_{j-1}^n}^{t_j^n} \{A(\omega) \, u_n(s, \omega) - f_n(s, \omega)\} \, ds \\
&= v(\omega) - \int_0^t A(\omega) \, u_n(s, \omega) \, ds + \int_0^t f_n(s, \omega) \, ds.
\end{aligned}$$

Letting $n \to \infty$, we have

$$u(t, \omega) = v(\omega) - \int_0^t A(\omega) \, u(s, \omega) \, ds + \int_0^t f(s, \omega) \, ds.$$

Thus u is a solution of the random differential equation in consideration. If w: $[0, T] \times \Omega \to X$ is another solution, then, since for each fixed $\omega \in \Omega$, $\| u(\cdot, \omega) - w(\cdot, \omega)\|$ is absolutely continuous, we have for a.e. $t \in [0, T]$

$$\begin{aligned}
&d \| u(t, \omega) - w(t, \omega)\|/dt \\
&\quad = [u(t, \omega) - w(t, \omega), du(t, \omega)/dt - dw(t, \omega)/dt]_- \\
&\quad = [u(t, \omega) - w(t, \omega), -A(\omega) \, u(t, \omega) + A(\omega) \, w(t, \omega)]_- \\
&\quad \leqslant -[u(t, \omega) - w(t, \omega), (A(\omega) + k(\omega)I) \, u(t, \omega) - (A(\omega) + k(\omega)I) \, w(t, \omega)]_+ \\
&\qquad + [u(t, \omega) - w(t, \omega), k(\omega)(u(t, \omega) - w(t, \omega)]_+ \\
&\quad \leqslant k(\omega) \, \| u(t, \omega) - w(t, \omega)\|.
\end{aligned}$$

Hence for any $t \in [0, T]$,

$$\| u(t, \omega) - w(t, \omega)\| \leqslant e^{k(\omega)t} \| u(0, \omega) - w(0, \omega)\| = 0.$$

Therefore this u is the desired unique solution. Q.E.D.

By the uniqueness of a solution on any finite interval we immediately obtain the following corollary.

COROLLARY 2.4. *Let X be a separable Banach space, A: $\Omega \times X \to X$ be a continuous random operator and f: $[0, \infty) \times \Omega \to X$ be a mapping satisfying condition (C, Ω). Suppose there exists a function k: $\Omega \to R$ such that* $\sup\{k(\omega)$: $\omega \in \Omega\} = M < \infty$ *and for any $\omega \in \Omega$, $A(\omega) + k(\omega)I$ is accretive. Then for each measurable mapping v: $\Omega \to X$, there exists a unique mapping u: $[0, \infty) \times \Omega \to X$ satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $u(0, \omega) = v(\omega)$ and*

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) = f(t, \omega) \qquad (t \geqslant 0).$$

3. Random Differential Equations in Hilbert Spaces

The following theorem is a special case of Theorem 4.3 in [18], but we give the elementary proof for its importance.

Let E be a finite dimensional Banach space and $D = \{x \in E: \| x - y \| \leqslant r\}$, where $y \in E$ and $r > 0$.

THEOREM 3.1. *Let f: $[0, T] \times D \times \Omega \to E$ be a mapping with the following properties*:

(i) *For any $\omega \in \Omega$, $f(\cdot, \cdot, \omega)$ is jointly continuous*;

(ii) *For any $t \in [0, T]$ and $x \in D$, $f(t, x, \cdot)$ is measurable*;

(iii) $M = \sup\{\|f(t, x, \omega)\|: 0 \leqslant t \leqslant T,\ x \in D,\ \omega \in \Omega\} < \infty$.

Let v: $\Omega \to D$ be a measurable mapping for which $\sup\{\| v(\omega) - y \|: \omega \in \Omega\} = r_1 < r$, *and* $T_1 = \min\{T, (r - r_1)/M\}$. *Then there exists a mapping u: $[0, T_1] \times \Omega \to D$ satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $u(0, \omega) = v(\omega)$ and*

$$du(t, \omega)/dt = f(t, u(t, \omega), \omega) \qquad (0 \leqslant t \leqslant T_1).$$

Proof. Denote $K = \{h \in C([0, T_1], E): \| h(t) - y \| \leqslant r$ for all $t \in [0, T_1]\}$, and define W: $\Omega \times K \to K$ by

$$W(\omega, h)(t) = v(\omega) + \int_0^t f(s, h(s), \omega)\, ds$$

($\omega \in \Omega$, $0 \leqslant t \leqslant T_1$, $h \in K$), then

$$\| W(\omega, h)(t) - y \| \leqslant \| v(\omega) - y \| + \int_0^{T_1} \| f(s, h(s), \omega) \| \, ds$$
$$\leqslant r_1 + T_1 M \leqslant r.$$

It is not difficult to show that for each fixed $h \in K$, $W(\omega, h)(t)$ is measurable in ω and continuous in t. Hence $W(\cdot, h)$ is measurable as a mapping from Ω into K by [18, Proposition 4.2]. It is well-known that for each $\omega \in \Omega$, $W(\omega, \cdot)$ is a compact mapping from K into K (cf. Edwards [11, pp. 164–165]). By [18, Corollary 2.2] there exists a measurable mapping $u\colon \Omega \to K$ such that for any $\omega \in \Omega$, $W(\omega, u(\omega)) = u(\omega)$. By [18, Proposition 4.2] again, u may be considered as a mapping u: $[0, T_1] \times \Omega \to D$ satisfying condition (C, Ω). It follows that

$$u(t, \omega) = v(\omega) + \int_0^t f(s, u(s, \omega), \omega) \, ds \qquad (0 \leqslant t \leqslant T_1, \omega \in \Omega),$$

hence for this u we have the desired conclusion. Q.E.D.

Now we show the existence of a solution for a random differential equation with time dependent accretive operators in a Hilbert space. This is a stochastic analogue of Browder [6, Theorem 4] (cf. Kato [22, Theorem 2] and Vainberg [26, Theorem 26.1]).

THEOREM 3.2. *Let H be a separable Hilbert space and A: $[0, T] \times \Omega \times H \to H$ be a mapping having the following properties*:

(i) *For each $\omega \in \Omega$, $A(\cdot, \omega)(\cdot)$ is jointly continuous*;

(ii) *For each bounded subset D of H,* $\sup\{\| A(t, \omega)x \| : 0 \leqslant t \leqslant T, \omega \in \Omega, x \in D\} < \infty$;

(iii) *For each $t \in [0, T]$ and $x \in H$, $A(t, \cdot)x$ is measurable*;

(iv) *There exists a function k: $[0, T] \times \Omega \to R$ such that* $\sup\{k(t, \omega): 0 \leqslant t \leqslant T, \omega \in \Omega\} = M < \infty$ *and for each $t \in [0, T]$ and $\omega \in \Omega$, $A(t, \omega) + k(t, \omega)I$ is monotone.*

Then for each measurable mapping $v\colon \Omega \to H$, there exists a unique mapping u: $[0, T] \times \Omega \to H$ satisfying condition (C_1, Ω) such that for any $\omega \in \Omega$, $u(0, \omega) = v(\omega)$ and

$$du(t, \omega)/dt + A(t, \omega)\, u(t, \omega) = 0 \qquad (0 \leqslant t \leqslant T).$$

Proof. By introducing the transformation $u(t, \omega) \to e^{-Mt} u(t, \omega)$, we may assume that $k(t, \omega) = 0$ for all $t \in [0, T]$ and $\omega \in \Omega$, that is $A(t, \omega)$ is monotone. Let $Y = L^2([0, T], H)$ be the L^2-space of H-valued function on $[0, T]$. Let L' be a linear operator on Y defined by $(L'h)(t) = dh(t)/dt$ with domain $D(L') = \{h \in C_1([0, T], H): h(0) = 0\}$. The operator L' has a closed linear extension L

on Y with domain $D(L) \subset C([0, T], H)$ dense in Y. L is maximal monotone and has a continuous inverse L^{-1}, where

$$(L^{-1}g)(t) = \int_0^t g(s)\, ds$$

with domain $D(L^{-1}) = Y$ (cf. Kato [22, Lemma A1] and Vainberg [26, Lemma 26.1]).

(I) First we assume that $v(\omega) = y \in H$ for all $\omega \in \Omega$. Define $G: \Omega \times C([0, T], H) \to Y$ by $(G(\omega)h)(t) = B(t, \omega)\, h(t)$, where $B(t, \omega)x = A(t, \omega)(x + y)(h \in C([0, T], H)$, $0 \leqslant t \leqslant T, x \in H$, $\omega \in \Omega$). Then, for each $\omega \in \Omega$, $G(\omega)$ maps bounded sets in $C([0, T], H)$ into bounded sets in Y and $G(\omega)h \in C([0, T], H)$ for any $h \in C([0, T], H)$. Moreover, if $\{h_n\}$ converges to h in $C([0, T], H)$, then $\{G(\omega)h_n\}$ converges weakly to $G(\omega)h$ in Y, that is $G(\omega)$ is demicontinuous (cf. Kato [22, Lemma A2] and Vainberg [26, Lemma 26.2]). If $h \in C([0, T], H)$, then for any $t \in [0, T]$, the mapping $\omega \to (G(\omega)h)(t)$ is measurable, hence $G(\cdot)h$ is measurable as a mapping from Ω to $C([0, T], H)$ by [18, Proposition 4.2]. Thus $G(\cdot)h$ is measurable as a mapping from Ω to Y. Choose a countable complete orthonormal system $\{e_n\}$ of H. For each n ($n = 1, 2,\ldots$), let H_n be the n-dimensional subspace of H spanned by $\{e_1, \ldots, e_n\}$ and P_n be the orthogonal projection of H onto H_n. We need the following lemma.

LEMMA 3.3. *There exists a mapping $v_n : [0, T] \times \Omega \to H_n$ satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $v_n(0, \omega) = 0$ and*

$$dv_n(t, \omega)/dt = -P_nB(t, \omega)\, v_n(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

Proof of Lemma 3.3. Let $r_0 = \| y \|$, then by (ii) $\sup\{\| A(t, \omega)y \|: 0 \leqslant t \leqslant T, \omega \in \Omega\} = C < \infty$. Denote $r_1 = \max\{r_0, TC\}$ and take $r > r_1$. By Theorem 3.1 there exists a mapping $x_1 : [0, T_1] \times \Omega \to D$ satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $x_1(0, \omega) = 0$ and

$$dx_1(t, \omega)/dt = -P_nB(t, \omega)\, x_1(t, \omega) \qquad (0 \leqslant t \leqslant T_1),$$

where $D = \{x \in H_n : \| x \| \leqslant r\}$ and T_1 is as in Theorem 3.1. Then for $t \in [0, T_1]$ and $\omega \in \Omega$,

$$\| x_1(t, \omega)\| \leqslant \int_0^T \| B(t, \omega)0 \|\, dt$$
$$= \int_0^T \| A(t, \omega)y \|\, dt \leqslant TC \leqslant r_1.$$

Let $T_2 = \min\{T, 2T_1\}$ and $K_1 = \{h \in C([T_1, T_2], H_n)$: $\| h(t)\| \leqslant r$ for all $t \in [T_1, T_2]\}$. Define $W_1 : \Omega \times K_1 \to K_1$ by

$$W_1(\omega, h)(t) = x_1(T_1, \omega) - \int_{T_1}^t P_nB(s, \omega)\, h(s)\, ds,$$

then as in the proof of Theorem 3.1, W_1 is a compact random operator. By [18, Corollary 2.2] there exists a measurable mapping $x_2 : \Omega \to K_1$ such that $W_1(\omega, x_2(\omega)) = x_2(\omega)$ for all $\omega \in \Omega$. Using [18, Proposition 4.2], x_2 may be considered as a mapping from $[T_1, T_2] \times \Omega$ into D satisfying condition (C, Ω). We have $x_2(T_1, \omega) = x_1(T_1, \omega)$ and

$$dx_2(t, \omega)/dt = -P_n B(t, \omega)\, x_2(t, \omega) \qquad (t \in [T_1, T_2], \omega \in \Omega).$$

Define $y_1 : [0, T_2] \times \Omega \to D$ by $y_1(t, \omega) = x_1(t, \omega)$ if $0 \leqslant t \leqslant T_1$, or $x_2(t, \omega)$ if $T_1 \leqslant t \leqslant T_2$ and $\omega \in \Omega$, then y_1 satisfies condition (C_1, Ω) and $y_1(0, \omega) = 0$,

$$dy_1(t, \omega)/dt = -P_n B(t, \omega)\, y_1(t, \omega) \qquad (\omega \in \Omega, 0 \leqslant t \leqslant T_2).$$

Repeating this procedure finite times, we obtain a desired solution v_n. Q.E.D.

We continue the proof of Theorem 3.2. For this v_n, $\| v_n(t, \omega)\| \leqslant r_1$ $(0 \leqslant t \leqslant T, \omega \in \Omega)$. Each v_n may be considered as a mapping of Ω into $C([0, T], H)$. By (ii) there exists $C_1 > 0$ such that $\|(G(\omega)\, v_n(\omega))(t)\| \leqslant C_1$ $(0 \leqslant t \leqslant T, \omega \in \Omega, n = 1, 2,...)$. Hence for some $C_2 > 0$, we have $\| G(\omega)\, v_n(\omega)\|_2 \leqslant C_2$ $(\omega \in \Omega, n = 1, 2,...)$, where $\| \cdot \|_2$ is the norm of Y. Since for any $\omega \in \Omega$, $G(\omega)$ is demicontinuous, $G(\cdot)\, v_n(\cdot)$ is measurable as a mapping of Ω into Y for any n by a similar proof as in Kannan and Salehi [21, Theorem VI (2.19)]. For each n, define $F_n: \Omega \to 2^Y$ (the family of all subsets of Y) by $F_n(\omega) = \text{w-cl}\{-G(\omega)\, v_i(\omega): i \geqslant n\}$ for $\omega \in \Omega$, where w-cl(Z) is the weak closure of Z. Then, as in the proof of [18, Theorem 2.5], there exists a measurable mapping $z: \Omega \to Y$ such that for fixed $\omega \in \Omega$, some subsequence $\{-G(\omega)\, v_m(\omega)\}$ of $\{-G(\omega)\, v_n(\omega)\}$ converges weakly to $z(\omega)$ in Y. Then $\{Lv_m(\omega)\} = \{-P_m G(\omega)\, v_m(\omega)\}$ converges weakly to $z(\omega)$ in Y. Since L^{-1} is continuous, $\{v_m(\omega)\}$ converges weakly to $L^{-1}z(\omega) = v_0(\omega)$. v_0 is a measurable mapping of Ω into Y. Moreover, L^{-1} is continuous as a mapping of Y into $C([0, T], H)$, hence v_0 is measurable as a mapping of Ω into $C([0, T], H)$. By [18, Proposition 4.2] v_0 may be considered as a mapping of $[0, T] \times \Omega$ into H satisfying condition (C, Ω), thus $v_0(0, \omega) = 0$ for all $\omega \in \Omega$. As in the deterministic case, for any $\omega \in \Omega$, we obtain $Lv_0(\omega) + G(\omega)\, v_0(\omega) = 0$, hence $v_0(\omega) + L^{-1}G(\omega)\, v_0(\omega) = 0$. Define u: $[0, T] \times \Omega \to H$ by $u(t, \omega) = v_0(t, \omega) + y$, then we have

$$u(t, \omega) + \int_0^t A(s, \omega)\, u(s, \omega)\, ds = y \qquad (0 \leqslant t \leqslant T, \omega \in \Omega).$$

For this u the conclusion holds.

(II) Now we assume that $v: \Omega \to H$ is any measurable mapping. For any $x \in H$, let $u_x : [0, T] \times \Omega \to H$ be a solution of $u_x(0, \omega) = x$ and

$$du_x(t, \omega)/dt + A(t, \omega)\, u_x(t, \omega) = 0 \qquad (0 \leqslant t \leqslant T, \omega \in \Omega)$$

by (I). Then for any $x, y \in H$,

$$\begin{aligned} d(\| u_x(t, \omega) - u_y(t, \omega)\|^2)/dt &= 2(du_x(t, \omega)/dt - du_y(t, \omega)/dt, u_x(t, \omega) - u_y(t, \omega)) \\ &= 2(-A(t, \omega)\, u_x(t, \omega) + A(t, \omega)\, u_y(t, \omega), u_x(t, \omega) - u_y(t, \omega)) \\ &\leqslant 0, \end{aligned}$$

hence

$$\begin{aligned} \| u_x(t, \omega) - u_y(t, \omega)\| &\leqslant \| u_x(0, \omega) - u_y(0, \omega)\| \\ &= \| x - y \|. \end{aligned}$$

Thus, $u_x(t, \omega)$ is uniquely defined and continuous in x. Define u: $[0, T] \times \Omega \to H$ by $u(t, \omega) = u_{v(\omega)}(t, \omega)$ $(0 \leqslant t \leqslant T,\ \omega \in \Omega)$, then $u(t, \omega)$ is measurable in ω by Himmelberg [16, Theorem 6.5], and $u(0, \omega) = u_{v(\omega)}(0, \omega) = v(\omega)$ for all $\omega \in \Omega$.

This u is a desired mapping. The uniqueness follows from a similar inequality as above. This completes the proof of Theorem 3.2. Q.E.D.

We easily deduce the following corollary.

COROLLARY 3.4. *Let H be a separable Hilbert space and A: $[0, \infty) \times \Omega \times H \to H$ be a mapping with the following properties*:

(i) *For any $\omega \in \Omega$, $A(\cdot, \omega)(\cdot)$ is jointly continuous*;

(ii) *For any bounded subset D of H and $T > 0$, $\sup\{\| A(t, \omega)x \|: 0 \leqslant t \leqslant T, \omega \in \Omega, x \in D\} < \infty$*;

(iii) *For any $t \geqslant 0$ and $x \in H$, $A(t, \cdot)x$ is measurable*;

(iv) *There exists a function k: $[0, \infty) \times \Omega \to R$ such that for any $T > 0$, $\sup\{k(t, \omega): 0 \leqslant t \leqslant T, \omega \in \Omega\} < \infty$ and for any $t \geqslant 0$, $\omega \in \Omega$, $A(t, \omega) + k(t, \omega)I$ is monotone.*

Then for any measurable mapping v: $\Omega \to H$, there exists a unique mapping u: $[0, \infty) \times H \to H$ satisfying condition (C_1, Ω) such that for any $\omega \in \Omega$, $u(0, \omega) = v(\omega)$ and

$$du(t, \omega)/dt + A(t, \omega)\, u(t, \omega) = 0 \qquad (t \geqslant 0).$$

4. PERIODIC SOLUTIONS OF RANDOM DIFFERENTIAL EQUATIONS

By using existence theorems in sections 3 and 4, we obtain periodic solutions for various random differential equations in Banach or Hilbert spaces. For corresponding deterministic results, we refer to Brézis [5], Barbu [1] and their references.

THEOREM 4.1. *Let X be a separable Banach space, A: $\Omega \times X \to X$ be a* *continuous random operator and* $f: [0, T] \times \Omega \to X$ *be a mapping satisfying condition* (C, Ω). *Suppose there exists a function* $k: \Omega \to (-\infty, 0)$ *such that for each* $\omega \in \Omega$, $A(\omega) + k(\omega)I$ *is accretive. Then there exists a unique mapping* u: $[0, T] \times \Omega \to X$ *satisfying condition* (C_1, Ω) *such that for each* $\omega \in \Omega$, $u(0, \omega) = u(T, \omega)$ *and*

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

Proof. By Theorem 2.3, for any $x \in X$, there exists a unique mapping u_x : $[0, T] \times \Omega \to X$ satisfying condition (C_1, Ω) such that for any $\omega \in \Omega$, $u_x(0, \omega) = x$ and

$$du_x(t, \omega)/dt + A(\omega)\, u_x(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

Fix $\omega \in \Omega$. If $x, y \in X$, then since $\| u_x(\cdot, \omega) - u_y(\cdot, \omega)\|$ is absolutely continuous, it is differentiable a.e. $t \in [0, T]$ and

$$d(\| u_x(t, \omega) - u_y(t, \omega)\|)/dt \leqslant k(\omega)\, \| u_x(t, \omega) - u_y(t, \omega)\|$$

(cf. the proof of Theorem 2.3). Thus, for any $t \in [0, T]$,

$$\begin{aligned} \| u_x(t, \omega) - u_y(t, \omega)\| &\leqslant e^{k(\omega)}\, \| u_x(0, \omega) - u_y(0, \omega)\| \\ &= e^{k(\omega)}\, \| x - y \|. \end{aligned}$$

Define $g: \Omega \times X \to X$ by $g(\omega, x) = u_x(T, \omega)$, then by the above inequality g is a contraction random operator. By Hanš [12], there exists a unique measurable mapping $z: \Omega \to X$ such that $g(\omega, z(\omega)) = z(\omega)$ for all $\omega \in \Omega$. Let u: $[0, T] \times \Omega \to X$ be a unique mapping satisfying condition (C_1, Ω) such that for any $\omega \in \Omega$, $u(0, \omega) = z(\omega)$ and

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T)$$

by Theorem 2.3. It follows that $u(T, \omega) = u_{z(\omega)}(T, \omega) = g(\omega, z(\omega)) = z(\omega) = u(0, \omega)$ for every $\omega \in \Omega$. If v: $[0, T] \times \Omega \to X$ is another periodic solution, then for any $t \in [0, T]$, $\omega \in \Omega$,

$$\| u(t, \omega) - v(t, \omega)\| \leqslant e^{k(\omega)}\, \| u(0, \omega) - v(0, \omega)\|,$$

in particular

$$\| u(T, \omega) - v(T, \omega)\| \leqslant e^{k(\omega)}\, \| u(0, \omega) - v(0, \omega)\|.$$

Since $k(\omega) < 0$ and $\| u(T, \omega) - v(T, \omega)\| = \| u(0, \omega) - v(0, \omega)\|$, the above inequality implies that $u(0, \omega) = v(0, \omega)$ $(\omega \in \Omega)$, hence $u(t, \omega) = v(t, \omega)$ $(0 \leqslant t \leqslant T, \omega \in \Omega)$. Q.E.D.

COROLLARY 4.2. *Let* X *be a separable Banach space,* A: $\Omega \times X \to X$ *be a* *continuous accretive random operator,* $f\colon [0, T] \times \Omega \to X$ *be a mapping satisfying condition* (C, Ω) *and* $\lambda\colon \Omega \to (0, \infty)$ *be a measurable function. Then there exists a unique mapping* $u\colon [0, T] \times \Omega \to X$ *satisfying condition* (C_1, Ω) *such that for any* $\omega \in \Omega$, $u(0, \omega) = u(T, \omega)$ *and*

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) + \lambda(\omega)\, u(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

Proof. Let $B\colon \Omega \times X \to X$ be a mapping defined by $B(\omega)x = A(\omega)x + \lambda(\omega)x$ ($\omega \in \Omega$, $x \in X$), then B is a continuous random operator and for any $\omega \in \Omega$, $B(\omega) - \lambda(\omega)I$ is accretive. We obtain the desired solution by Theorem 4.1.

Q.E.D.

THEOREM 4.3. *Let* H *be a separable Hilbert space and* $A\colon [0, T] \times \Omega \times H \to H$ *be a mapping with conditions* (i), (ii), (iii) *in Theorem* 3.2 *and the following*:

(iv) *There exists a function* $k\colon [0, T] \times \Omega \to (-\infty, 0)$ *such that for any* $t \in [0, T]$ *and* $\omega \in \Omega$, $A(t, \omega) + k(t, \omega)I$ *is monotone and for any* $\omega \in \Omega$, $k(\cdot, \omega)$ *is measurable with* $-\infty < M(\omega) < 0$, *where*

$$M(\omega) = \int_0^T k(t, \omega)\, dt.$$

Then there exists a unique mapping $u\colon [0, T] \times \Omega \to H$ *satisfying condition* (C_1, Ω) *such that for any* $\omega \in \Omega$, $u(0, \omega) = u(T, \omega)$ *and*

$$du(t, \omega)/dt + A(t, \omega)\, u(t, \omega) = 0 \qquad (0 \leqslant t \leqslant T).$$

Proof. By Theorem 3.2, for each $x \in H$, there exists a unique mapping u_x : $[0, T] \times \Omega \to H$ satisfying condition (C_1, Ω) such that for any $\omega \in \Omega$, $u_x(0, \omega) = x$ and

$$du_x(t, \omega)/dt + A(t, \omega)\, u_x(t, \omega) = 0 \qquad (0 \leqslant t \leqslant T).$$

Let $\omega \in \Omega$ be fixed. If $x, y \in H$, then for $t \in [0, T]$,

$$d(\| u_x(t, \omega) - u_y(t, \omega)\|^2)/dt \leqslant 2k(t, \omega)\, \| u_x(t, \omega) - u_y(t, \omega)\|^2$$

(cf. the proof of Theorem 3.2), thus we have

$$\begin{aligned}\| u_x(t, \omega) - u_y(t, \omega)\| &\leqslant \exp\left(\int_0^t k(s, \omega)\, ds\right) \| u_x(0, \omega) - u_y(0, \omega)\| \\ &= \exp\left(\int_0^t k(s, \omega)\, ds\right) \| x - y \|,\end{aligned}$$

in particular

$$\begin{aligned}\| u_x(T, \omega) - u_y(T, \omega)\| &\leqslant \exp\left(\int_0^T k(t, \omega)\, dt\right) \| x - y \| \\ &= e^{M(\omega)}\, \| x - y \|.\end{aligned}$$

Let $g: \Omega \times H \to H$ be a mapping defined by $g(\omega, x) = u_x(T, \omega)$, then g is a contraction random operator. By Hanš [12] there exists a unique measurable mapping $z: \Omega \to H$ such that $g(\omega, z(\omega)) = z(\omega)$ for all $\omega \in \Omega$. Let u: $[0, T] \times \Omega \to H$ be a unique mapping satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $u(0, \omega) = z(\omega)$ and

$$du(t, \omega)/dt + A(t, \omega)\, u(t, \omega) = 0 \qquad (0 \leqslant t \leqslant T).$$

By the same way as in the proof of Theorem 4.1, this u is the unique periodic solution. Q.E.D.

The same reasoning implying Corollary 4.2 from Theorem 4.1 yields the following corollary.

COROLLARY 4.4. *Let H be a separable Hilbert space and A: $[0, T] \times \Omega \times H \to H$ be a mapping satisfying conditions* (i), (ii), (iii) *in Theorem* 3.2 *and the following*:

(iv) *For each $t \in [0, T]$ and $\omega \in \Omega$, $A(t, \omega)$ is monotone.*

Let λ: $\Omega \to (0, \infty)$ be a measurable mapping for which $\sup\{\lambda(\omega)$: $\omega \in \Omega\} < \infty$. *Then there exists a unique mapping u: $[0, T] \times \Omega \to H$ satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $u(0, \omega) = u(T, \omega)$ and*

$$du(t, \omega)/dt + A(t, \omega)\, u(t, \omega) + \lambda(\omega)\, u(t, \omega) = 0 \qquad (0 \leqslant t \leqslant T).$$

We extend [18, Theorem 2.6] to the following which is used to show the periodic solutions of random differential equations in Hilbert spaces other than those considered above.

PROPOSITION 4.5. *Let H be a separable Hilbert space, K be a nonempty bounded closed convex subset of H and $f: \Omega \times K \to H$ be a nonexpansive random operator for which $f(\omega, \partial K) \subset K$ for every $\omega \in \Omega$, where ∂K is the boundary of K. Then there exists a measurable mapping $z: \Omega \to K$ such that $f(\omega, z(\omega)) = z(\omega)$ for all $\omega \in \Omega$.*

Proof. Let $P: H \to K$ be the (metric) projection and define $g: \Omega \times K \to K$ by $g(\omega, x) = Pf(\omega, x)$, then g is a nonexpansive random operator. By [18, Theorem 2.6] there exists a measurable mapping $z: \Omega \to K$ such that $g(\omega, z(\omega)) = z(\omega)$ for any $\omega \in \Omega$. It is not difficult to observe that $f(\omega, z(\omega)) = z(\omega)$ for all $\omega \in \Omega$. Q.E.D.

Now periodic solutions of random differential equations in Hilbert spaces are given under similar conditions to those considered by Browder [7] in the deterministic case (cf. Brézis [5]).

THEOREM 4.6. *Let H be a separable Hilbert space, A: $\Omega \times H \to H$ be a continuous monotone random operator and $f: [0, T] \times \Omega \to H$ be a mapping* *satisfying condition* (C, Ω). *Suppose there exists* $M > 0$ *such that* $(A(\omega)x - f(t, \omega), x) > 0$ *whenever* $\omega \in \Omega$, $t \in [0, T]$ *and* $x \in H$ *with* $\| x \| = M$. *Then there exists a mapping* u: $[0, T] \times \Omega \to H$ *satisfying condition* (C_1, Ω) *such that for any* $\omega \in \Omega$, $u(0, \omega) = u(T, \omega)$ *and*

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

Proof. Let $K = \{x \in H: \| x \| \leqslant M\}$, then by Theorem 2.3 for each $x \in K$, there exists a unique mapping $u_x : [0, T] \times \Omega \to H$ satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $u_x(0, \omega) = x$ and

$$du_x(t, \omega)/dt + A(\omega)\, u_x(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

Define $g: \Omega \times K \to H$ by $g(\omega, x) = u_x(T, \omega)$. Since for $\omega \in \Omega$ and $t \in [0, T]$,

$$d(\| u_x(t, \omega)\|^2)/dt = 2(-A(\omega)\, u_x(t, \omega) + f(t, \omega), u_x(t, \omega)),$$

by hypothesis it follows that $d(\| u_x(t, \omega)\|^2)/dt < 0$ if $\| u_x(t, \omega)\| = M$. Hence, $\| u_x(T, \omega)\| \leqslant M$ whenever $\omega \in \Omega$ and $\| x \| = M$, that is $g(\omega, \partial K) \subset K$ for any $\omega \in \Omega$. Moreover, if $x, y \in K$ and $\omega \in \Omega$, then

$$\| g(\omega, x) - g(\omega, y)\| = \| u_x(T, \omega) - u_y(T, \omega)\| \leqslant \| x - y \|$$

(cf. the proof of Theorem 3.2). By Proposition 4.5 there exists a measurable mapping $z: \Omega \to K$ such that $g(\omega, z(\omega)) = z(\omega)$ for all $\omega \in \Omega$. Let u: $[0, T] \times \Omega \to H$ be a unique mapping satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $u(0, \omega) = z(\omega)$ and

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

Then for any $\omega \in \Omega$, we have $u(T, \omega) = u_{z(\omega)}(T, \omega) = g(\omega, z(\omega)) = z(\omega) = u(0, \omega)$. Q.E.D.

It is easy to observe that the following corollary holds.

COROLLARY 4.7. *Let* H *be a separable Hilbert space*, A: $\Omega \times H \to H$ *be a continuous monotone random operator and* f: $[0, T] \times \Omega \to H$ *be a mapping satisfying condition* (C, Ω) *for which* $\sup\{\| f(t, \omega)\|\, 0 \leqslant t \leqslant T,\ \omega \in \Omega\} < \infty$. *Suppose there exists a function* c: $(0, \infty) \to R$ *such that*

$$(A(\omega)x, x) \geqslant c(\| x \|)\, \| x \| \qquad (\omega \in \Omega, x \in H \text{ with } x \neq 0)$$

and $c(r) \to \infty$ *as* $r \to \infty$. *Then there exists a mapping* u: $[0, T] \times \Omega \to H$ *satisfying condition* (C_1, Ω) *such that for any* $\omega \in \Omega$, $u(0, \omega) = u(T, \omega)$ *and*

$$du(t, \omega)/dt + A(\omega)\, u(t, \omega) = f(t, \omega) \qquad (0 \leqslant t \leqslant T).$$

The following theorem can be proved by the same way as the proof of Theorem 4.6 by using Theorem 3.2 instead of Theorem 2.3, hence we omit the proof.

THEOREM 4.8. *Let H be a separable Hilbert space and A*: $[0, T] \times \Omega \times H \to H$ *be a mapping having properties* (i), (ii), (iii) *and* (iv) *in Corollary* 4.4. *Suppose there exists $M > 0$ such that $(A(t, \omega)x, x) > 0$ if $t \in [0, T]$, $\omega \in \Omega$ and $\| x \| = M$. Then there exists a mapping u: $[0, T] \times \Omega \to H$ satisfying condition (C_1, Ω) such that for any $\omega \in \Omega$, $u(0, \omega) = u(T, \omega)$ and*

$$du(t, \omega)/dt + A(t, \omega)\, u(t, \omega) = 0 \qquad (0 \leqslant t \leqslant T).$$

We have the following corollary.

COROLLARY 4.9. *Let H be a separable Hilbert space and A*: $[0, T] \times \Omega \times H \to H$ *be a mapping satisfying conditions* (i), (ii), (iii) *and* (iv) *in Corollary* 4.4. *Suppose there exists a function c*: $(0, \infty) \to R$ *such that*

$$(A(t, \omega)x, x) \geqslant c(\| x \|)\, \| x \|$$

whenever $t \in [0, T]$, $\omega \in \Omega$ and $x \in H$ with $x \neq 0$ and $c(r) \to \infty$ as $r \to \infty$. Then there exists a mapping u: $[0, T] \times \Omega \to H$ satisfying condition (C_1, Ω) such that for each $\omega \in \Omega$, $u(0, \omega) = u(T, \omega)$ and

$$du(t, \omega)/dt + A(t, \omega)\, u(t, \omega) = 0 \qquad (0 \leqslant t \leqslant T).$$

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